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Bounds on the thermal stresses in axisymmetrically heated spheres are developed.

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BOUNDS ON THERMAL STRESSES IN SPHERES

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Introduction

It is often easy to secure estimates for the highest and lowest temperatures to be expected in a structure, but it is much more difficult to obtain detailed information on the temperature distribution itself, short of a complete solution of the problem. It has been found possible, and quite useful, in many cases, to determine the maximum possible thermal stress which may develop, solely on the basis of the known bounds on the temperature. Upper and lower bounds on the thermal stresses and deformations have been constructed in this way for beams and plates [1] and for composite structures [2,3,4]. Applications of these bounds have been developed in the estimate of errors in approximate calculations [5] and in the analysis of thermal reduction of beam torsional rigidity [6], as well as, of course, in many practical design estimates. The present paper develops similar bounds for the thermal stresses which may arise in solid or hollow spheres. The temperature distributions considered are axisymmetric, and are otherwise either arbitrary or radially monotonic.

Analysis

For a hollow sphere of inner radius a and outer radius b with an axially symmetric temperature variation $T(r)$, the radial stress σ_{rr}, the circumferential stresses $\sigma_{\theta\theta}$ and $\sigma_{\phi\phi}$ and the displacement u are [7]:

$$\sigma_{rr} = \frac{2K}{1-\beta^3} \{(1 - \frac{\beta^3}{\rho^3}) \int_\rho^1 T\rho^2 d\rho - (\frac{1}{\rho^3} - 1) \int_\beta^\rho T\rho^2 d\rho\} \tag{1a}$$

$$\sigma_{\theta\theta} = \sigma_{\phi\phi} = \frac{K}{1-\beta^3} \{(\frac{1}{\rho^3} + 2) \int_\beta^\rho T\rho^2 d\rho + (\frac{\beta^3}{\rho^3} + 2) \int_\rho^1 T\rho^2 d\rho - (1 - \beta^3)T\} \tag{1b}$$

$$u = \frac{Kb}{2\mu(1-\beta^3)} \left\{\frac{\beta^3}{\rho^2}\int_\rho^1 T\rho^2 d\rho + \frac{1}{\rho^2}\int_\beta^\rho T\rho^2 d\rho + \frac{2\rho(1-2\nu)}{1+\nu}\int_\beta^1 T\rho^2 d\rho\right\} \tag{1c}$$

where $\beta = \frac{a}{b}$, $\rho = \frac{r}{b}$, $K = \frac{E}{1-\nu}$; E is Young's modulus, μ the shear modulus, Poisson's ratio and α the coefficient of thermal expansion.

Assume now that bounds on the temperature distribution are known, i.e.

$$T_m \leq T(r) \leq T_M \tag{2}$$

We can then write, from equation (1a),

$$\sigma_{rr}(\rho) \leq \frac{2K}{1-\beta^3}\left\{(1 - \frac{\beta^3}{\rho^3})(\frac{1-\rho^3}{3})T_M - (\frac{1}{\rho^3} - 1)(\frac{\rho^3-\beta^3}{3})T_m\right\}$$

and therefore a bound for σ_{rr} at any given value of ρ is:

$$|\sigma_{rr}(\rho)| \leq \frac{2K}{1-\beta^3}\frac{(1-\rho^3)(\rho^3-\beta^3)}{3\rho^3}(T_M - T_m) \tag{3}$$

The expression modifying $(T_M - T_m)$ is positive and assumes a maximum at $\rho^2 = \beta$, and so in the interval $\beta \leq \rho \leq 1$:

$$|\sigma_{rr}| \leq \frac{2K}{3}\frac{1-\beta^{3/2}}{1+\beta^{3/2}}(T_M - T_m) \tag{4}$$

For the case of a thin shell, $\tau = \frac{b-a}{b}$ is small, and equation (4) can be written as:

$$|\sigma_{rr}| \leq K(T_M - T_m)(\frac{\tau}{2} + \frac{\tau^2}{4} + \frac{7}{96}\tau^3 + \ldots) \tag{5}$$

Similarly, we obtain from equations (1b) and (1c) the bounds:

$$|\sigma_{\theta\theta}| \leq K(T_M - T_m) \tag{6a}$$

$$\alpha a T_m \leq u \leq \alpha b T_M \tag{6b}$$

Monotonic Temperature Distributions

When the temperature increases monotonically with r, then $T_m = T(\beta)$ and $T(1) = T_M$, and the lower bound on σ_{rr} can be shown to be zero since, from equation (1a), we have:

$$\sigma_{rr} \geq \frac{2K}{1-\beta^3} \{T(\rho)\frac{1-\rho^3}{3}(1 - \frac{\beta^3}{\rho^3}) - T(\rho)(\frac{1}{\rho^3} - 1)\frac{\rho^3-\beta^3}{3}\} = 0$$

so that

$$0 \leq \sigma_{rr} \leq \frac{2}{3}\frac{1-\beta^{3/2}}{1+\beta^{3/2}} K[T(1) - T(\beta)] \qquad (7)$$

and as before

$$|\sigma_{\theta\theta}| \leq K[T(1) - T(\beta)] \qquad (8)$$

$$\alpha a T(\beta) \leq u \leq \alpha b T(1) \qquad (9)$$

Stresses at the Edges

For an arbitrary temperature distribution, the circumferential stresses at the inner and outer edges of the sphere are, respectively:

$$\sigma_{\theta\theta}(\beta) = \frac{K}{1-\beta^3} \{3 \int_\beta^1 T\rho^2 d\rho - (1 - \beta^3)T(\beta)\}$$

$$\sigma_{\theta\theta}(1) = \frac{K}{1-\beta^3} \{3 \int_\beta^1 T\rho^2 d\rho - (1 - \beta^3)T(1)\}$$

Hence,

$$\sigma_{\theta\theta}(1) = \sigma_{\theta\theta}(\beta) + K[T(\beta) - T(1)] \qquad (10)$$

Using the relation in equation (8), we conclude that, for a monotonically increasing temperature distribution, $\sigma_{\theta\theta}$ (1) is a compressive stress and $\sigma_{\theta\theta}$ (β) is a tensile stress. The reverse holds for a monotonically decreasing temperature distribution. Note that $T(\beta)$ and $T(1)$ cannot be equal unless T is constant throughout.

Solid Sphere

For a solid sphere a = 0, and equations (1) reduce to:

$$\sigma_{rr} = 2K\{\int_0^1 T\rho^2 d\rho - \frac{1}{\rho^3}\int_0^\rho T\rho^2 d\rho\}$$

$$\sigma_{\theta\theta} = \sigma_{\phi\phi} = K\{\frac{1}{\rho^3}\int_0^\rho T\rho^2 d\rho + 2\int_0^1 T\rho^2 d\rho - T\} \tag{11}$$

$$u = \frac{Kb}{2\mu}\{\frac{1}{\rho^2}\int_0^\rho T\rho^2 d\rho + 2\rho\frac{1-2\nu}{1+\nu}\int_0^1 T\rho^2 d\rho\}$$

Again, with $T_m \leq T(r) \leq T_M$, we can write the bounds as:

$$|\sigma_{rr}| \leq (2/3)K[T_M - T_m]$$

$$|\sigma_{\theta\theta}| \leq K[T_M - T_m] \tag{12}$$

$$\alpha T_m \leq u \leq \alpha T_M$$

and for a monotonically increasing temperature distribution:

$$0 \leq \sigma_{rr} \leq \frac{2K}{3}(1 - \rho^3)[T(1) - T(0)] \leq \frac{2}{3}K[T(1) - T(0)]$$

$$-K[T(1) - T(0)] \leq \sigma_{\theta\theta} \leq \frac{2}{3} K[T(1) - T(0)] \tag{13}$$

$$\alpha T(0) \leq u \leq \alpha T(1)$$

It can be noted that for the solid sphere

$$\sigma_{rr}(0) = \sigma_{\theta\theta}(0) = 2K \{ \int_0^1 T\rho^2 d\rho - \frac{T(0)}{3} \} \tag{14}$$

and the equilibrium condition [7]

$$\frac{d\theta_{rr}}{dr} = \frac{2}{r} (\sigma_{rr} - \sigma_{\theta\theta}) \text{ yields } \frac{d\sigma_{rr}}{dr} = 0 \tag{15}$$

when $r = 0$. However, for the hollow sphere, $\sigma_{rr}(\beta) \equiv 0$ and

$$\lim_{\beta \to 0} \sigma_{\theta\theta}(\beta) = K \{ 3 \int_0^1 T\rho^2 d\rho - T(0) \} \tag{16}$$

consequently, $\frac{d\sigma_{rr}}{dr}$ becomes infinite as $\beta \to 0$.

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